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Hidden Convexity in Active Learning: A Convexified Online Input Design for ARX Systems

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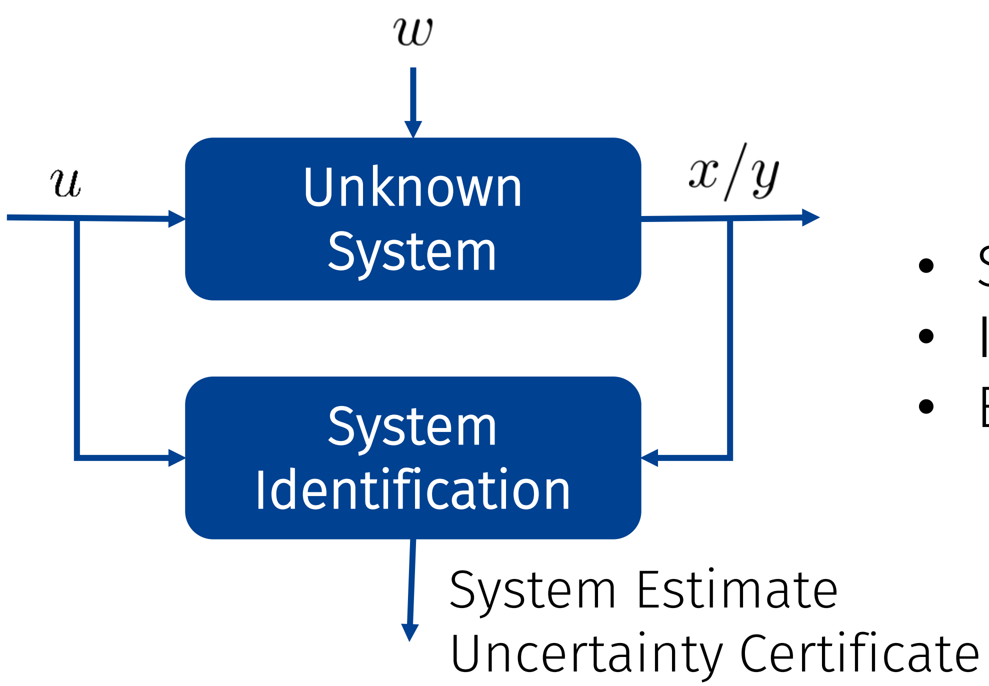
System Identification requires informative data



- System identification is central for data-driven control
- Informative data required (PE)
- Excitation u drives information content

How should u be selected to improve identification?

On experiment design

	Asymptotic Regime	Finite Sample Regime
Guarantees	Convergence	Sample Complexity
System classes	State-Space, Frequency Domain, ARX	State-Space **ARX**
Optimization problem	Non-convex, Convex reformulations exist	Non-convex **Convex Reformulation**

Contribution

Computationally tractable experiment design scheme for **ARX systems** with **sample complexity guarantees**

Outline

Active Learning Algorithm

Adaptation to ARX Systems

Convex Reformulation

Finite Sample Experiment Design

We build on the sequential algorithm in [1]

- Finite sample guarantees
- E-optimal input design using **periodic input** sequences
  - Optimizes Fourier coefficients of input
- Predicts future behaviour of covariates using **OLS** and **CE**

$$\check{u}^* \in \arg\max_{\{\check{u}_\ell\}_{\ell\in[k_i]}} \lambda_{\min}\left(\text{Predicted covariance} + \text{Past covariance} \right) \quad \text{(AL–OP)}$$

$$\text{s.t.} \quad \text{Bounded Input Energy}$$

Limitations

- Not applicable to I/O-data (only state-space systems)
- (AL–OP) is a non-convex optimization problem
  - Online solution requires an efficient solution
  - Guarantees only hold for global optimizer

[1] A. Wagenmaker and K. Jamieson. *Active Learning for Identification of Linear Dynamical Systems*. Conference on Learning Theory (2020)

Outline

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Convex Reformulation

State-space representation of ARX systems

ARX-Systems

- Unknown ARX-System

$$y_t = \sum_{i=1}^{p} A_i^* y_{t-i} + \sum_{j=1}^{q} B_j^* u_{t-j} + \Sigma_w^{\frac{1}{2}} w_t, \qquad \text{(ARX)}$$

  - Matrices $\{A_i^*\}_{i=1}^p$, $\{B_j^*\}_{j=1}^q$ are unknown (p, q are known)

- Define $x_t := \begin{bmatrix} y_{t-1:t-p}^\top & u_{t-1:t-q}^\top \end{bmatrix}^\top$, $\theta^* := \begin{bmatrix} A_{1:p}^* & B_{1:p}^* \end{bmatrix}$
(ARX) can be rewritten as

$$y_t = \theta^* x_t + \Sigma_w^{\frac{1}{2}} w_t$$

- Evolution of x_t satisfies[2]:

$$x_{t+1} = \mathcal{A}^* x_t + \mathcal{B}_u u_t + \mathcal{B}_w w_t$$

  - $\mathcal{A}^*$: Matrix containing all unknowns
  - $\mathcal{B}_u$: Known Matrix

- Estimate θ^* using OLS
- Predict covariance using state space description

➡ Finite sample guarantees:

$$\mathbb{P}\left[\|\hat{\theta}_T - \theta^*\| \leq \mathcal{O}\left(\frac{1}{\sqrt{T}}\right)\right] \geq 1 - \delta$$

[2] I. Ziemann, et al. *A tutorial on the non-asymptotic theory of system identification*. 62nd IEEE Conference on Decision and Control (CDC). IEEE, 2023.

Outline

Active Learning Algorithm

Adaptation to ARX Systems

Convex Reformulation

A closer look at the optimization problem (AL–OP)

$$\max_{\{\check{u}_\ell\}_{\ell \in [k_i]}} \lambda_{\min}\left(\frac{\gamma^2}{2} T_i \Gamma_{k_i}^{\check{u}}(\hat{A}, B) + \sum_{t=1}^{T} x_t x_t^\top\right)$$

$$\text{s.t.} \quad \sum_{\ell \in [k_i]} \mathrm{Tr}(\check{u}_\ell \check{u}_\ell^{\mathsf{H}}) \leq \frac{k_i^2 \gamma^2}{2}$$

$\Longleftrightarrow$

$$\max_{\{\check{U}_\ell\}_{\ell \in [k_i]}} \lambda_{\min}\left(\frac{\gamma^2}{2} T_i \Gamma_{k_i}^{\check{U}}(\hat{A}, B) + \sum_{t=1}^{T} x_t x_t^\top\right)$$

$$\text{s.t.} \quad \sum_{\ell \in [k_i]} \mathrm{Tr}(\check{U}_\ell) \leq \frac{k_i^2 \gamma^2}{2}$$

$$\mathrm{rank}(\check{U}_\ell) = 1 \quad \forall \ell \in [k_i]$$

non-convex

with

$$\Gamma_k^{\check{u}}(A, B) = \frac{1}{\gamma^2 k^2} \sum_{\ell=0}^{k-1} (e^{j\frac{2\pi\ell}{k}} I - A)^{-1} B \check{u}_\ell \check{u}_\ell^{\mathsf{H}} B^{\mathsf{H}} (e^{j\frac{2\pi\ell}{k}} I - A)^{-\mathsf{H}}$$

$$\Gamma_k^{\check{U}}(A, B) = \frac{1}{\gamma^2 k^2} \sum_{\ell=0}^{k-1} (e^{j\frac{2\pi\ell}{k}} I - A)^{-1} B \check{U}_\ell B^{\mathsf{H}} (e^{j\frac{2\pi\ell}{k}} I - A)^{-\mathsf{H}}$$

Convex reformulation

Theorem:

Let $\check{U}_\ell^*$ be the solution of the optimization problem

$$\max_{\{\check{U}_\ell\}_{\ell\in[k_i]}} \quad \lambda_{\min}\left(\frac{\gamma^2}{2} T_i \Gamma_{k_i}^{\check{U}}(\hat{A}, B) + \sum_{t=1}^{T} x_t x_t^\top\right)$$

$$\text{s. t.} \quad \sum_{\ell\in[k_i]} \mathrm{Tr}(\check{U}_\ell) \leq \frac{k_i^2\gamma^2}{2}.$$

Then the **principal eigenvectors** of $\check{U}_\ell^*, \ell \in [k_i]$ are the optimal solution of $(\mathrm{AL{-}OP})$

- Convex relaxation provides the optimal solution to the non-convex problem
- Holds for state-space and ARX systems
- Idea inspired by [3]

[3] I. Manchester. *Input design for system identification via convex relaxation*. 49th IEEE Conference on Decision and Control (CDC). IEEE, 2010.

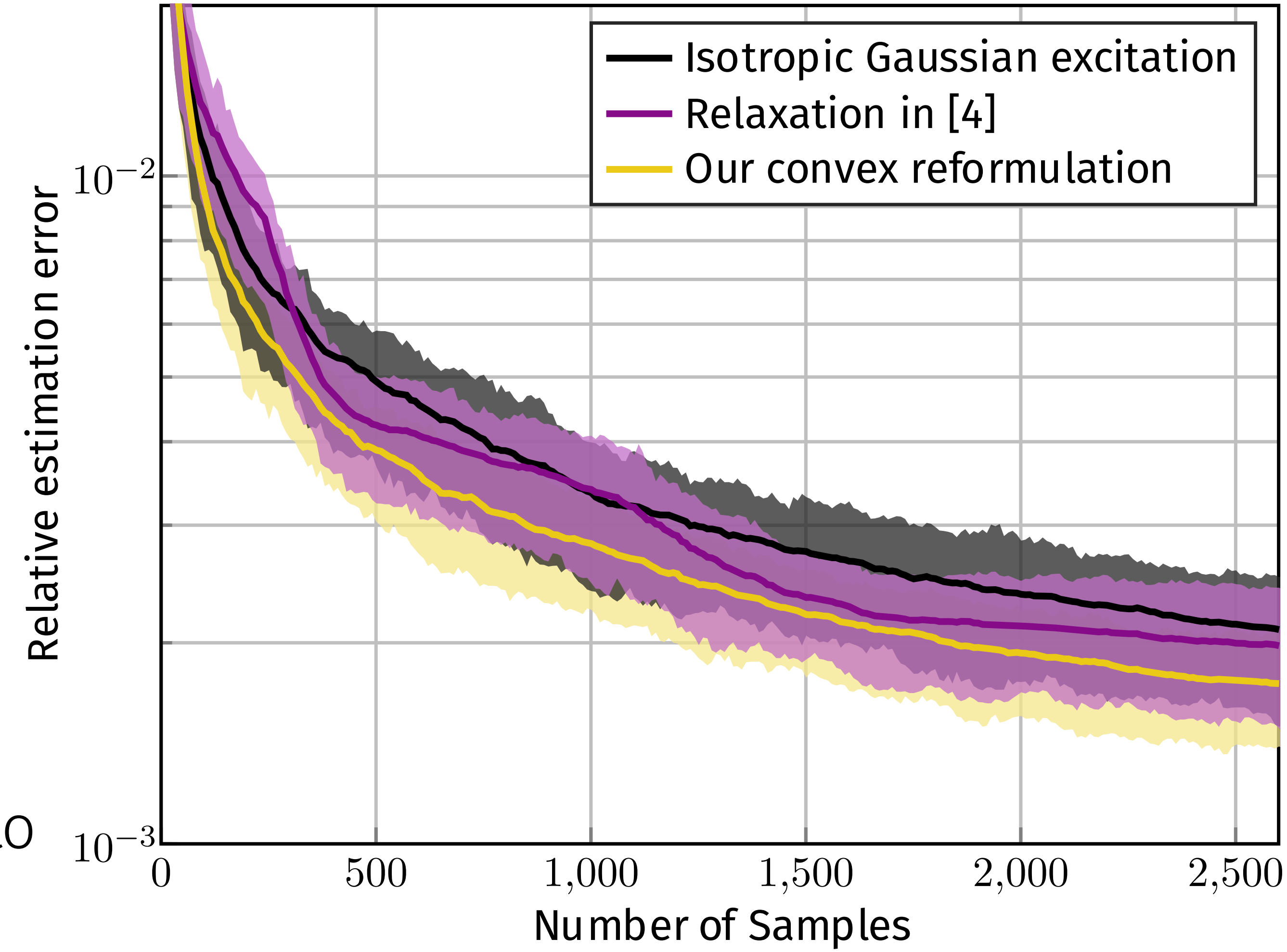
Numerical evaluation

- ARX system:

$$A_1^* = \begin{bmatrix} 0.7 & 0.1 \\ 0 & 0.9 \end{bmatrix}, \; A_2^* = \begin{bmatrix} -0.5 & 0 \\ 0.1 & -0.2 \end{bmatrix},$$

$$B_1^* = \begin{bmatrix} 0.1 & 0 \\ 0 & 5 \end{bmatrix}$$

- Comparison of different excitation strategies
  - Isotropic Gaussian
  - Convex relaxation by [4] (recovers optimal excitation asymptotically)
  - Our convex reformulation
- Improved performance compared to random excitation and [4]



[4] A. Wagenmaker, M. Simchowitz, K. Jamieson. *Task-optimal exploration in linear dynamical systems*. International Conference on Machine Learning. (2021).

Conclusion & Outlook

- Extension of Active Learning Algorithm in [1] to ARX-Systems
- Convex reformulation of active learning objective in [1]
  - Recovers the exact solution in a computationally tractable way

Future work

- Investigate benefit of experiment design[5]
- Finite sample algorithms for other experiment design criteria
- Strengthen the connection between the asymptotic and finite-sample regime

[1] A. Wagenmaker and K. Jamieson. *Active Learning for Identification of Linear Dynamical Systems*. Conference on Learning Theory (2020)
[5] **N. Chatzikiriakos**, K. Jamieson, and A. Iannelli. *High Effort, Low Gain: Fundamental Limits of Active Learning for Linear Dynamical Systems*. arXiv:2509.11907 (2025).

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Numerical evaluation (full)

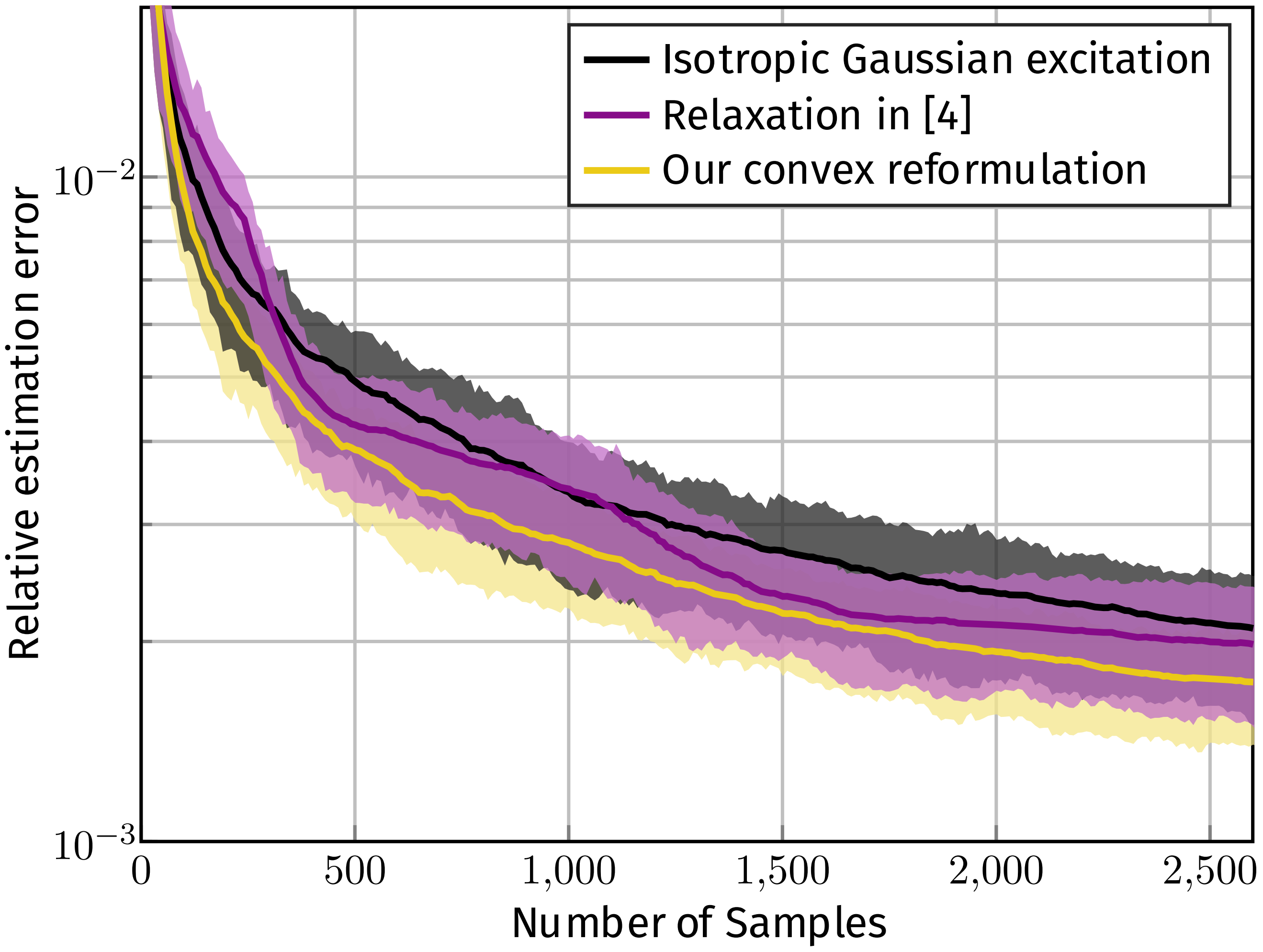
- ARX system:

$$A_1^* = \begin{bmatrix} 0.7 & 0.1 \\ 0 & 0.9 \end{bmatrix}, \quad A_2^* = \begin{bmatrix} -0.5 & 0 \\ 0.1 & -0.2 \end{bmatrix},$$

$$B_1^* = \begin{bmatrix} 0.1 & 0 \\ 0 & 5 \end{bmatrix} \qquad \Sigma_w = I$$

- Mean & 25% and 75% percentiles of estimation error over 200 MC runs

- Comparison of different excitation strategies
    - Isotropic Gaussian
    - Convex relaxation by [4] (recovers optimal excitation asymptotically)
    - Our convex reformulation



[4] A. Wagenmaker, M. Simchowitz, K. Jamieson. *Task-optimal exploration in linear dynamical systems*. International Conference on Machine Learning. (2021).